A Benders Decomposition Approach for an Integrated Bin Allocation and Vehicle Routing Problem in Municipal Waste Management

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Abstract. The municipal solid waste system is a complex reverse logistic chain which comprises several optimisation problems. Although these problems are interdependent – i.e., the solution to one of the problems restricts the solution to the other – they are usually solved sequentially in the related literature because each is usually a computationally complex problem. We address two of the tactical planning problems in this chain by means of a Benders decomposition approach: determining the location and/or capacity of garbage accumulation points, and the design of collection routes for vehicles. We also propose a set of valid inequalities to speed up the resolution process. Our approach manages to solve medium-sized real-world instances in the city of Bahía Blanca, Argentina, showing smaller computing times in comparison to solving a full MIP model.

Keywords: municipal solid waste; reverse supply chain; integrated allocation-routing problem; Benders decomposition algorithm; valid inequalities.

1 Introduction

Regardless of their size, city councils have the duty to provide efficient service to their constituents. Municipal Solid Waste (MSW) management is a crucial example of such a service. It has direct economic and social impacts; poor collection service can be both expensive and unsanitary. In this paper, we will focus on a less traditional MSW design called Garbage Accumulation Points (GAP). Instead of providing a "door-to-door" pickup of garbage, constituents have to drop their garbage at specific facility – the GAPs. These facilities can range from collective bins to recycling centres. When using GAPs, MSW management comprises the following design decisions:

– The design of a pre-collection network, which consists in defining the location and capacity of GAPs.

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– The design and schedule of routes for collection vehicles.

The geographical distribution of GAPs affects the actual route that the collection vehicles must perform. Additionally, the storage capacity of these sites will define the visit frequency in order to avoid overflow. Finally, the availability and type of vehicles[1] affect the distribution and capacity of the GAPs in a (global) optimal solution. Thus, there is a trade-off between the cost of the installation of GAPs and the routing cost; solving both simultaneously is often beneficial [12]. However, solutions in the literature [7,10,20] often address each separately. This is due to the complexity of tackling MSW as a whole. Indeed, only solving the design of routes is tantamount to solving a Vehicle Routing Problem (VRP) [23], a well-known NP-hard problem.

In this paper, we propose the following contributions to the field of MSW management with GAPs:

– A novel mathematical model which combines the allocation of bin arrangements to GAPs and defining collection routes (Section 3.1).
– A Benders decomposition-based approach to tackle the resulting problem (Section 4).

The problem we are tackling is an "inventory routing problem" [4]. The resulting formulation is a mixed-integer program (MIP) which is still too large to be tractable. However, we can see it as a combination of two problems:

1. a routing problem, similar to a vehicle routing problem [23]; and,
2. an allocation problem, similar to a nonlinear resource allocation problem [3] – in which the used amount of resource (bin) should be minimized, though not limited.

This *natural* decomposition lead us to use Benders decomposition [1], a well-suited method for problems with this structure. Benders decomposition works by solving such problems in an iterative fashion. First, it solves the *difficult* part to generate a candidate solutions. It then checks this solution against the dual of the *easy* part. From the dual solution, it either terminates, when the dual solution's objective value is equal to an incumbent; or, it generates constraints, called "Benders cuts," which are added to the difficult part and the problem solved anew.

However, we cannot use standard Benders decomposition because the subproblem contains integer variables. Therefore, we use a framework called Unified branch-and-Benders-cut (UB&BC) [17]. This framework is based on a modified Branch-and-Cut (B&C) with callbacks from a commercial solver. In the callbacks, it derives dual information and an upper bound for the subproblem. Using these, it terminates the branch-and-bound tree with a set of *open solutions* – whose objective function value falls below the best upper bound. To find the global optimum, the B&C is followed by a post-processing phase where the framework solves those open solutions to integer optimality.

[1] Mainly capacity, but could also be cost.

We test our model on a real-world use case: the city of Bahía Blanca, Argentina (Section 6). Although the city currently uses a door-to-door collection service, they are interested in switching to GAPs. We simulated instances using data from a survey [5] and provide optimal allocation and routing for a variety of scenarios. Finally, we present our conclusions and future directions in Section 7.

2 A working example

In this section we will present a short example to illustrate the workings of our solution approach. We will use a toy instance shown in Fig. 1a. It comprises: a two-day-horizon, two GAPs, and two vehicles. We also consider two types of waste bins with a storage capacities of 1.1m^3 and 1.73m^3 respectively.

We will now show the iterations the solution algorithm takes.[2] At each iteration, we will report:

- the master solution (routing cost, which includes GAP allocation per vehicle per day);
- the objective function value of the LP relaxation of the subproblem (lower bound);
- the heuristic value (upper bound); and,
- the total cost of the solution.

A graphical representation of the iterations is provided in Figs. 1b to 1d.

Iteration 1. The first solution uses one vehicles on two days and one vehicle the second day only.

$$\begin{aligned} v_{0,0} &: (0,2) \rightarrow (2,0) \\ v_{0,1} &: (0,2) \rightarrow (2,0) \\ v_{1,0} &: (0,1) \rightarrow (1,0) \end{aligned}$$

The routing cost is: 494.2. The LP relaxation has an objective function value of 7.96 while the heuristic has a value of 10.48. We add a Benders cut to the master problem and continue.

Iteration 2. The second solution uses two vehicles with different routes during one day:

$$\begin{aligned} v_{0} &: (0,2) \rightarrow (2,0) \\ v_{1} &: (0,1) \rightarrow (1,0) \end{aligned}$$

The routing cost is: 381.8. The LP relaxation has an objective function value of 7.06 while the heuristic has a value of 10.48. We add a Benders cut to the master problem and continue.

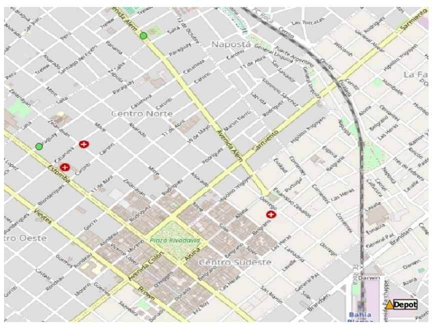
[2] We use the complete problem (**M1**) augmented with valid inequalities Eqs. (2) to (4). The Benders cuts we generate are "optimality cuts" given by Eq. (8b).

Iteration 3. The third solution found uses a single vehicle with the same route on both days, given by:

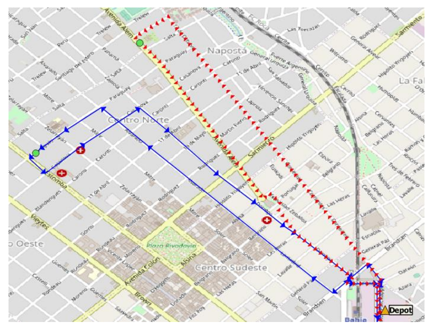
$$v_0 : (0, 1) \rightarrow (1, 2) \rightarrow (2, 0)$$

The routing cost is: 316.8. The LP relaxation has an objective function value of 8.58 while the heuristic has a value of 10.48. We add a Benders cut to the master problem and continue.

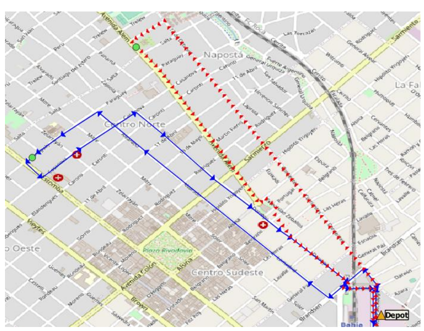
At this point, the B&C will finish as no improving solution can be found, we can progress to the post-processing.



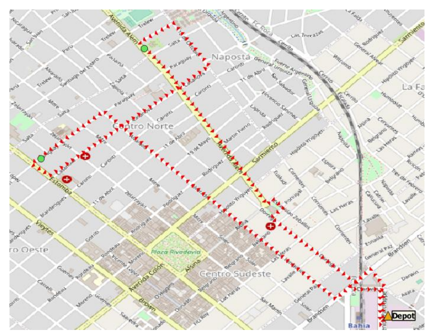
(a) Location of the depot and the two GAPs (green circles) on the toy instance.



(b) It. 1: The first (blue) vehicle uses its route both days, while the second (red) vehicle only operates on the first day.



(c) It. 2: Both vehicles operate during the first day.



(d) It. 3: Only one vehicle operates during one day

Post-processing. At the start of the post-processing phase, the UB&BC orders solutions according to their lower bound values. In this case, it will process the solutions in reverse order: 3, 2, 1.

Starting with the solution found in Iteration 3, we solve the subproblem to integer optimality. This gives an optimal value of 10.48 – the same as the heuristic. Being an integer value, it can be used to update the upper bound to: 347.28 (routing + delivery).

Then, the framework verifies that the remaining open solutions' values are lower that the new-found upper bound. Both solutions found at Iterations 1 and 2 exceed the best upper bound and are thus skipped.

Our approach has managed to find the optimal solution to the problem. It did so as an integrated algorithm which solved the routing and allocation problems at once.

3 A mathematical model of MSW

3.1 Model formulation

The model has the following sets:

- I: the set of potential GAPs.
- $L = \{l_0, l_1, \ldots, l_{|L|}\}$: is an ordered set of vehicles. We consider a homogeneous and finite fleet of vehicles.
- T: the set of days in the time horizon, which coincides with a week (seven days).
- R: the set of possible visit combinations.
- U: the set of all bin arrangements that can be installed in a GAP.

A potential GAP $i \in I$ is a predefined location in an urban area in which bins can be installed. We define the superset: $I^0 = I \cup 0$, where 0 is the depot from which vehicles start and finish their daily tours, and where the collected waste is deposited. We also define a special notation for the set of edges given a set of nodes: $E(\cdot)$, such that: $E(I) = \{\, (i, j) \mid i \in I, j \in I, i \neq j \,\}$. Bin arrangements are set of bins that are feasible to install in a GAP, respecting the space limitation.

We now define the parameters of the model:

- Q: vehicle capacity.
- c_{ig}: travel time between i to g.
- s_i: service time of GAP i.
- b_i: waste generation per day at GAP i.
- cap_u: capacity of bin arrangement u.
- cin_u: adjusted cost of installing bin arrangement u for the time horizon T.
- α: cost per kilometre of transportation.
- β_r: maximum number of days between two consecutive visits of the visit combination r.
- a_{rt}: 1 if day t is included in visit combination r.
- TL: time limit of the working day.

Notice that cin_u is an *adjusted cost*. This is because we are considering two different level of decision and cost:

1. a strategic decision that involves purchasing and installing the bin arrangements that will last probably for several years; and,
2. a tactical decision which involves the transport costs of the routing schedule [19].

Therefore, the cost assigned to a bin arrangement (cin_u) includes a proportional part of the purchase/installation cost and the maintenance cost. With regards to parameters a_{rt} and β_r, they can be introduced with an example:

Example 1. Let time horizon T be a week – i. e., $T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$. Then, one possible visit combination $r* \in R$ is $\{t_1, t_3, t_5, t_7\}$. In this case, we have: $a_{r*t_1} = a_{r*t_3} = a_{r*t_5} = a_{r*t_7} = 1$, and, conversely: $a_{r*t_2} = a_{r*t_4} = a_{r*t_6} = 0$. Thus, the maximum number of days between two consecutive visits that this combination has is two days: $\beta_{r*} = 2$, and the chosen bin arrangement for this GAP must be able to store the waste generated in two days. (A similar consideration is performed in [12].)

Finally, we define the following decision variables:

- x_{iglt}: binary variable set to 1 if vehicle l performs the collection route between GAPs i and g on day t, 0 otherwise.
- v_{iglt}: continuous variable representing the load of vehicle l along the path between GAP i and g on day t.
- m_{ir}: binary variable set to 1 if visit combination r is assigned to GAP i, 0 otherwise.
- n_{ui}: binary variable set to 1 if bin arrangement $u \in U$ is used for GAP i, 0 otherwise.

We can now present the mathematical model for the MSW management problem:

$$\min \quad \sum_{\substack{i \in I \\ u \in U}} n_{ui}\ cin_u + \alpha \sum_{i,g \in E(I^0)} c_{ig} \left(\sum_{\substack{l \in L \\ t \in T}} x_{iglt} \right) \tag{\textbf{M1}}$$

$$s.t. \quad \sum_{u \in U} n_{ui}\ cap_u \geq \sum_{r \in R} b_i m_{ir} \beta_r \qquad \forall\ i \in I \tag{1a}$$

$$\sum_{u \in U} n_{ui} = 1 \qquad \forall\ i \in I \tag{1b}$$

$$\sum_{r \in R} m_{ir} = 1 \qquad \forall\ i \in I \tag{1c}$$

$$\sum_{\substack{g \in I^0, g \neq i \\ l \in L}} x_{iglt} - \sum_{r \in R} a_{rt} m_{ir} = 0 \qquad \forall\ t \in T,\ i \in I \tag{1d}$$

$$\sum_{i \in I^0, i \neq q} x_{iqlt} - \sum_{g \in I^0, g \neq q} x_{qglt} = 0 \qquad \forall\ q \in I^0,\ l \in L,\ t \in T \tag{1e}$$

$$\sum_{i \in I} x_{0ilt} \leq 1 \qquad \forall\ l \in L,\ t \in T \tag{1f}$$

$$\sum_{i,g \in E(I^0)} (c_{ig} + s_i)\ x_{iglt} \leq TL \qquad \forall\ l \in L,\ t \in T \tag{1g}$$

$$v_{iglt} \leq Q\ x_{iglt} \qquad \forall\ (i,j) \in E(I^0), l \in L,\ t \in T \tag{1h}$$

$$\sum_{i\in I^0, i\neq g} v_{iglt} + b_g \sum_{r\in R} (m_{gr}\ \beta_r) \leq \sum_{i\in I^0, i\neq g} v_{gilt} + Q\ \left(1 - \sum_{i\in I^0, i\neq g} x_{iglt}\right) \quad \forall\ g\in I,\ l\in L,\ t\in T \qquad \text{(1i)}$$

$$v \geq 0; n, x, b \in \mathbb{B}$$

The objective function is the sum of the routing cost and the adjusted cost of installing bins. Equation (1a) limits the maximum amount of garbage that can be accumulated in a GAP to the installed capacity of the bin arrangement. Equation (1b) enforces that one bin arrangement has to be chosen for each GAP. Equation (1c) establishes that one visit combination is assigned to each GAP. Equation (1d) ensures that each GAP is visited by the collection vehicle the days that corresponds to the assigned visit combination. Equation (1e) ensures that if a vehicle visits a GAP, it leaves the GAP on the same day. Equation (1f) states that every vehicle can be used at most once a day. Equation (1g) guarantees that a tour does not last longer than the allowable time limit associated with the working day of the drivers. Equation (1h) limits the total amount of waste collected in a tour to the vehicle capacity. Equation (1i) establishes that the outbound flow after visiting a GAP equals the inbound flow plus the waste collected from that GAP and, thus, also forbids subtours.

3.2 Valid inequalities

The model presented above for the MSW (**M1**) is still a difficult problem. In particular, it contains a lot of *symmetric solutions.* Two solutions are said to be symmetric if they have the same objective function value but different variable assignments. Consider the following: during a given day, two trucks undertaking the same collection route would have the same cost. There is no way for the solver to omit one of them.

One way to address this issue is to add Valid Inequalities (VIs) to the model. A VI is a constraint that reduces the feasible polytope of the problem without removing every optimal solution. We decided to focus on VIs for the routing part of the problem because the allocation part is *easy* in comparison. For examples of VIs in the context of vehicle routing problems, we refer the interested reader to [6].

One thing to remember is that our graph is asymmetric. Therefore, we do not need to address symmetries in routes with the same GAPs. We have developed the following valid inequalities to remove as much symmetry from the optimal solutions as possible.

Empty start. A vehicle must start its tour unloaded. This prevents solutions with different *delivery plans* – when a vehicle finishes its collection tour below full capacity, we can consider another solution where the vehicle starts with any amount less than the difference.

$$v_{0glt} = 0, \forall\ g \in I,\ l \in L,\ t \in T \qquad (2)$$

Vehicle ordering. We impose that a vehicle with index l can only leave the depot if the vehicle with index $l-1$ has. In the case where a solution does not use all available vehicles, we can consider swapping an unused vehicle with a used one. For brevity, we define: $L' = L \setminus \{0\}$, as the set of vehicles minus the first one.

$$\sum_{i \in I} x_{0ilt} \leq \sum_{i \in I} x_{0ipt}, \forall\ l \in L',\ p = l-1,\ t \in T \tag{3}$$

Furthest visit. We assign the furthest GAP from the depot to the first vehicle. Because each GAP must be visited at most once a day, so does the furthest. Because only one vehicle can visit each GAP on a given day, we can forbid others vehicle than the first vehicle – using L' defined above – to visit the furthest GAP.

$$\sum_{i \in I, t \in T} x_{iglt} = 0, \forall\ l \in L', g = \operatorname*{argmax}_{i \in I} c_{0i} \tag{4}$$

4 A resolution approach based on Benders decomposition

The classic Benders decomposition was devised in [1] for addressing large MIPs that have a characteristic *block diagonal* structure. In summary, it starts by decomposing the *original problem* into a *master problem* and a *subproblem.* The master problem is a relaxation of the original problem used to determine the values of a subset of its variables. It is formed by retaining the *complicating variables*, and projecting out the other variables and replacing them with an *incumbent.* The subproblem is formed around the projected variables and a parameterised version of the complicating variables. By enumerating the extreme points and rays of the subproblem, the algorithm defines the projected costs and the feasibility requirements, respectively, of the complicating variables. Because this enumeration is seldom tractable, the algorithm proceeds in the following manner:

1. It solves the (relaxed) master problem to optimality, which yields a *candidate solution.*
2. This candidate solution is used as a parameter in the subproblem.
3. The resulting problem is solved to optimality and, using LP duality, a set of coefficients are retrieved.
4. These coefficients are used to generate a constraint, called a “Benders cut,” which is added to the master problem.
5. If the objective function value of the subproblem is equal to the incumbent value in the master problem, the algorithm stops. Otherwise, it repeats from point 1. using the master problem with the additional constraint.

One key limitation of the classic Benders decomposition is that the subproblem cannot contain integer variables. This is because of point 3. above: the method needs to use LP duality, which is not well-defined for MIPs. We use a recent framework called Unified branch-and-Benders-cut (UB&BC) [18] to bypass this issue. This new framework operates by using a modified B&C where, at each integer node, it:

1. solves the LP relaxation of the subproblem to get a lower bound and generate Benders cuts; and,
2. uses a heuristic to determine if the master solution is feasible and, if yes, a valid, global upper bound.

The second point is key: by maintaining a valid upper bound, the framework ensures that no optimal solution is removed during the search. However, this leads to having a set of *open solutions* after finishing the B&C tree – solutions whose objective function value falls between the lower and upper bound. Thus, the UB&BC finishes by a *post-processing phase* during which subproblems associated with open solutions are solved to integer optimality. The combination of maintaining a global upper bound and using a post-processing phase enables the framework to find an optimal solution.

As stated in Section 3.1, the problem addressed in this work comprises two characteristic decision-making problems in MSW. On the one hand, the allocation of bins in the GAPs and, on the other, the design and schedule of routes for the collection vehicles. This division can be exploited by applying Benders decomposition. The bins allocation equations are moved to the subproblem while the master problem aims at designing the schedule and routes of the collection vehicles.

4.1 Creating the subproblem

The subproblem, which aims at allocating the bins of each GAP, is an integer programming problem:

$$q(\overline{m}) = \min \quad \sum_{\substack{i \in I \\ u \in U}} n_{ui}\ cin_u \tag{\textbf{SB}}$$

$$s.t. \quad \sum_{u \in U} n_{ui}\ cap_u \geq b_i \sum_{r \in R} \overline{m_{ir}}\ \beta_r \qquad \forall\ i \in I \tag{5a}$$

$$\sum_{u \in U} n_{ui} = 1 \qquad \forall\ i \in I \tag{5b}$$

$$n \in \mathbb{B}$$

We define the positive continuous variables δ_i and unrestricted continuous variables γ_i, the dual variables of Eqs. (5a) and (5b) respectively. The dual formulation of the LP relaxation of (**SB**), which will be used to generate cuts, is then:

$$q^{LP}(\overline{m}) = \max \quad \sum_{i \in I} \left(\gamma_i - \delta_i b_i \sum_{r \in R} (\overline{m_{ir}} \beta_r) \right) \tag{\textbf{LP}}$$

$$s.t. \quad \gamma_i - \delta_i \sum_{u \in U} cap_u \leq \sum_{u \in U} n_{ui} \qquad \forall i \in I \tag{6a}$$

$$\delta, \gamma \geq 0$$

Heuristic for the subproblem. In order to apply Benders decomposition when the subproblem has integer variables, an efficient method for solving the subproblem is required. Therefore, we devised a rounding heuristic procedure based on the LP relaxation of the subproblem:

1. We solve the LP relaxation of (**SB**). The (relaxed) solution will contain n_{ui} with fractional values.
2. We estimate the *joint fractional capacity* K_i^f of each GAP using:

$$K_i^f = \sum_{u \in U} n_{ui}\ cap_u \tag{7}$$

3. We define a feasible (non-fractional) bin arrangement $u \in U$ for each GAP by finding the bin arrangement with minimal cost among those with storage capacity larger than K_i^f. It is guaranteed that there will always be a bin arrangement which respects this rule since considering Eqs. (5b) and (7) implies that: $K_i^f \leq cap_{u*}, \forall\ i \in I$, where $u* = \mathrm{argmax}_{u \in U}\{cap_u\}$.

4.2 Stating the master problem

The master problem retains the same constraint structure as (**M1**) but the bin allocation part is replaced by an incumbent variable q. Let us consider the set of extreme points ($\mathcal{O}$) and extreme rays ($\mathcal{F}$) of the LP relaxation of (**SB**). These generate the optimality (8b) and feasibility (8a) cuts, respectively. Therefore, the master problem is:

$$\min \quad \alpha \sum_{i,g \in E(I^0)} c_{ig} \left(\sum_{\substack{l \in L \\ t \in T}} x_{iglt} \right) + q \tag{\textbf{MPB}}$$

$$s.t. \quad \text{Eqs. (1c) to (1i) and (2) to (4)}$$

$$\sum_{i \in I} \left(\gamma_i^f - \delta_i^f b_i \sum_{r \in R} (\beta_r m_{ir}) \right) \leq 0 \qquad \forall\ f \in \mathcal{F} \tag{8a}$$

$$\sum_{i \in I} \left(\gamma_i^f - \delta_i^o b_i \sum_{r \in R} (\beta_r m_{ir}) \right) \leq q \qquad \forall\ o \in \mathcal{O} \tag{8b}$$

$$v \geq 0; q \in \mathbb{R}; x \in \mathbb{B}$$

5 Literature review

Allocation of bins and routing problems have been thoroughly studied as separate problems in the MSW related literature [16]. Comprehensive reviews of the study of these problems separately can be found in [20] and [10], respectively. However, the number of works considering integrated approaches is more scarce.

Among the works that consider an integrated approach, [14] presented a study case of the Tunisian city of Sousse, considering uncertainty in waste generation at GAPs. They proposed a transformed formulation to handle stochastic waste generation and solved the problem in a heuristic fashion: first they applied the k-means clustering algorithm to group the GAPs into sectors and later they applied an exact model solved with CPLEX to determine both the number of bins and the collection route of each sector. They consider that all GAPs are to be collected daily.

Another example is [12], which proposed an integrated approach where the bins allocation problem is solved jointly with the routing schedule. The authors compare different methods to solve this problem based on a Variable Neighbourhood Search (VNS) algorithm, for solving the problem hierarchically – i. e., first solving the bin allocation and then the routing and *vice versa* – and integrated approaches. They found that integrated approaches overcome hierarchical ones.

A further complex approach is presented in [13] for solving an integrated model that aims to simultaneously locate GAPs, size the storage capacity of each GAP (allocate bins) and set the weekly collection schedule and routes in the context of collaborative recycling problem. They solved this problem with an Adaptive Large neighbourhood Search algorithm based on Hemmelmayr's implementation [11]. They performed a sensitivity analysis for several of the parameters, such as available vehicle capacities, visiting schedules or GAPs' storage capacities.

Although integrated models have proven to efficiently handle the trade-off [7, 9], others works have tackled these problems in a sequential fashion. For example, [8] solved the GAP location with while considering that bins of incompatible types – i. e., that cannot be collected by the same vehicle – are not located in the same GAP. Then they applied an heuristic zoning algorithm to define the routes while minimising the number of required vehicles and the total distance. Finally, [21] solved a multi-objective bins allocation problem in which one of the objectives was to minimise the required collection frequency.

Differently from the tactical problem that we address in this paper, other authors have used Bender's decomposition approaches to deal with optimization problems of the strategic level of the MSW logistic chain, mainly considering stochastic parameters (which is a traditional application area of Bender's decomposition). For example, [22] used Bender's decomposition to model a logistic chain of MSW in which organic waste in send from sources into treatment plants to generate power. Uncertainty is considered in waste generation, and power price and demand. Another case are [15], who applied Bender's decomposition to optimize the location and capacity selection of waste transfer stations when considering uncertainty in the operational cost of the stations.

6 Computational experiments

The instances used for these tests are based on simulated scenarios of the city of Bahía Blanca, Argentina. Although this city still has a door-to-door collection system, the local government and citizens are interested in studying more efficient collection systems that allow them to reduce the high logistic costs. In this sense, a community bins-based will simplified the required collection logistic [2, 5]. Particularly the location of 76 GAPs in a central neighbourhood of the city and the generation rate were obtained from a recent field work in a central neighbourhood of the city [5]. We consider a *homogeneous VRP* by using the standard collection vehicle of Bahía Blanca, a $20\,\mathrm{m}^3$ bin-tipper truck. The GAPs can hold three types of commercial bin with purchasing and maintenance cost (cin_u) 2.76, 3.53 and 5.24 monetary units (m.u.) and capacities (cap_u) 1.1, 1.73 and $3.1\,\mathrm{m}^3$, respectively. Information about the travel time between GAPs was estimated with Open Source Routing Machine[3] using the approach proposed by Vázquez Brust [24]. The algorithms are implemented in Python 3.5, and we use a UB&BC framework called BranDec[4] v0.7. For the benefit of the scientific community, we open-sourced the instances used for the experimentation[5] and implementation. The solver used is CPLEX v12.7 in its default configuration, we disable heuristics when running the UB&BC. We ran the experiments on a computer with Intel Gold 6148 Skylake CPU@2.4GHz and a 4GB RAM limit.

We divide the computational experimentation in two parts. In Section 6.1 we deal with small instances in order to asses the value of the proposed valid inequalities in the resolution approach. Then, in Section 6.2 we explore the performance of the proposed Benders approach in comparison to full MIP when solving more complex instances.

6.1 The value of valid inequalities

In order to explore the impact of valid inequalities in the resolution process we construct instances composed by five GAPs and the Depot. These sample of five GAPs were selected with QGIS Random Selection tool. We consider scenarios with two vehicles and two bin arrangements per GAP, the instances are formatted as "$I/T/n$," where n is the instance number. Figure 1 report the results of solving the resulting problem with:

MIP CPLEX using (**M1**);
MIP + VIs CPLEX using (**M1**) augmented with VIs (2) to (4);
BD our Benders approach; and,
BD + VIs our Benders approach augmented with VIs (2) to (4).

We ran five iterations of each configuration and report the minimum solve time. We can see in Fig. 1 that the VIs are necessary to have reasonable solve times. Both the MIP and our Benders approach benefit from them. When instances grow in size, that is when the time horizon is greater than two days, version with VIs do not manage to solve most instances. This experiment is not enough to tell for certain whether the Benders approach is better than MIP.

6.2 Using more bin arrangements

The complexity of the problem grows with the size of the instance becoming increasingly time consuming for the algorithm. Indeed, we now use a larger set of bin arrangements (set U) for the GAPs in order to have a more complex subproblem with a larger number of binary variables. In Fig. 2 we can see the

[3] http://project-osrm.org/
[4] `https://gitlab.com/Soha/brandec`
[5] http://doi.org/10.13140/RG.2.2.19210.49604

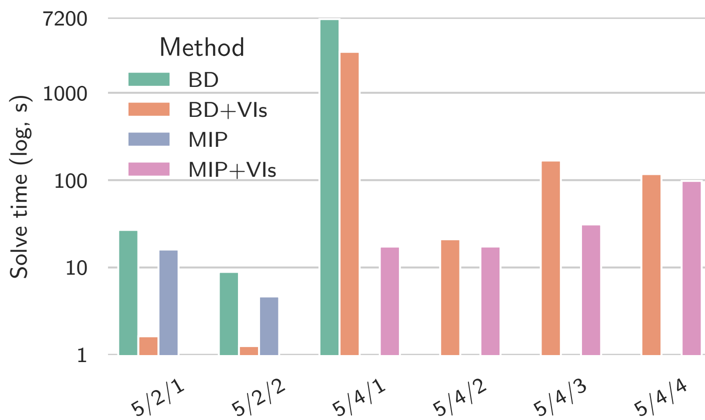


Fig. 1: Results of using different methods, with or without VIs, to solve a set of reduced instances.

time taken to find the optimal solution on a set of instances by our approach (BD) vs. solving the full MIP model with CPLEX (MIP). All the solutions have the valid inequalities (2) to (4) included. Our approach is, on average, more than one order of magnitude faster than using a full MIP. We are able to solve instances up to 7 GAPs, which the MIP cannot.

7 Conclusion

Municipal solid waste management is a critical issue in modern cities. Besides the direct environmental and social problems that can arise when it is mishandled, it usually represents a large portion of the municipal budgetary expense. Therefore, intelligent decision support tools that can efficiently provide this service to the citizens while also reducing the cost of the system can be a major asset for decision makers. This work addresses two common tactical problems that arise in the reverse logistic chain of solid waste: the design of a pre-collection network and the routing schedule of collection vehicles. These problems, which are usually solved individually in the related literature, are interdependent in the sense that the solution to one of the problem affects the other.

This work proposed an integrated approach that solves both problems simultaneously, making the trade-off an intrinsic element of the model. In particular, a new MIP formulation, valid inequalities and resolution approach based on Benders decomposition, using unified branch-and-Benders-cut, were proposed. Since the subproblem contains integer variables, we devised a heuristic for solving the bin allocation problem. Our approach was able to solve real-world instances in the city of Bahía Blanca. Tests performed on small instances showed the competitiveness of the valid inequalities to speed up the resolution process. Then, the resolution of larger instances showed that the proposed Benders approach was more competitive than full MIP. While not yet able to solve full-size instances, our approach holds promises to scale beyond a traditional MIP approach.

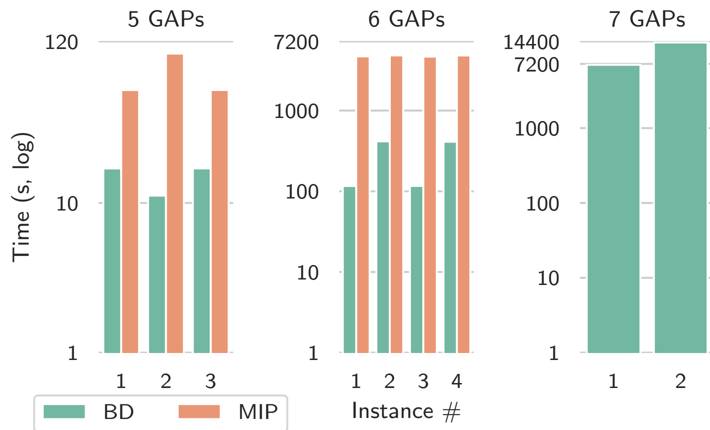


Fig. 2: Comparison of MIP vs. our Benders approach on a variety of scenarios. We report the time in log scale.

Future work includes expanding computational experiments with larger real-world instances to test the scalability of the approach. Another research line is to consider an allocation-first routing-second method. In that case, the master problem would be comparatively simpler than the subproblem. Such an approach would require efficient vehicle routing heuristics to work. We could also explore heterogeneous fleet of vehicles. Indeed, the city of Bahía Blanca already owns a fleet of vans of small capacity for spot operations.

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References

1. Benders, J.F.: Partitioning procedures for solving mixed-variables programming problems. Numerische mathematik 4(1), 238–252 (1962)
2. Bonomo, F., Durán, G., Larumbe, F., Marenco, J.: A method for optimizing waste collection using mathematical programming: a buenos aires case study. Waste Management & Research 30(3), 311–324 (2012)
3. Bretthauer, K.M., Shetty, B.: The nonlinear resource allocation problem. Operations research 43(4), 670–683 (1995)
4. Campbell, A., Clarke, L., Kleywegt, A., Savelsbergh, M.: The inventory routing problem. In: Fleet management and logistics, pp. 95–113. Springer (1998)
5. Cavallin, A., Rossit, D.G., Herrán Symonds, V., Rossit, D.A., Frutos, M.: Application of a methodology to design a municipal waste pre-collection network in real scenarios. Waste Management & Research 38(1), 117–129 (2020)

6. Dror, M., Laporte, G., Trudeau, P.: Vehicle routing with split deliveries. Discrete Applied Mathematics 50(3), 239–254 (1994)
7. Ghiani, G., Laganà, D., Manni, E., Musmanno, R., Vigo, D.: Operations research in solid waste management: A survey of strategic and tactical issues. Computers & Operations Research 44, 22–32 (2014)
8. Ghiani, G., Manni, A., Manni, E., Toraldo, M.: The impact of an efficient collection sites location on the zoning phase in municipal solid waste management. Waste management 34(11), 1949–1956 (2014)
9. Ghiani, G., Mourão, C., Pinto, L., Vigo, D.: Chapter 15: Routing in waste collection applications. In: Arc routing: problems, methods, and applications, pp. 351–370. SIAM (2015)
10. Han, H., Ponce Cueto, E.: Waste collection vehicle routing problem: literature review. PROMET-Traffic&Transportation 27(4), 345–358 (2015)
11. Hemmelmayr, V.C.: Sequential and parallel large neighborhood search algorithms for the periodic location routing problem. European Journal of Operational Research 243(1), 52–60 (2015)
12. Hemmelmayr, V.C., Doerner, K.F., Hartl, R.F., Vigo, D.: Models and algorithms for the integrated planning of bin allocation and vehicle routing in solid waste management. Transportation Science 48(1), 103–120 (2013)
13. Hemmelmayr, V.C., Smilowitz, K., de la Torre, L.: A periodic location routing problem for collaborative recycling. IISE Transactions 49(4), 414–428 (2017)
14. Jammeli, H., Argoubi, M., Masri, H.: A bi-objective stochastic programming model for the household waste collection and transportation problem: case of the city of sousse. Operational Research pp. 1–27 (2019)
15. Kŭdela, J., Šomplák, R., Nevrlỳ, V., Lipovskỳ, T., Smejkalová, V., Dobrovskỳ, L.: Multi-objective strategic waste transfer station planning. Journal of Cleaner Production 230, 1294–1304 (2019)
16. Lu, J.W., Chang, N.B., Liao, L., Liao, M.Y.: Smart and green urban solid waste collection systems: advances, challenges, and perspectives. IEEE Systems Journal 11(4), 2804–2817 (2015)
17. Mahéo, A.: Benders and its sub-problems. Ph.D. thesis, Australian National University, Canberra, Australia (2020)
18. Mahéo, A., Belieres, S., Adulyasak, Y., Cordeau, J.F.: Unified branch-and-Benders-cut for two-stage stochastic mixed-integer programs. Les Cahiers du GERAD G-2020-54 (2020)
19. Nagy, G., Salhi, S.: Location-routing: Issues, models and methods. European journal of operational research 177(2), 649–672 (2007)
20. Purkayastha, D., Majumder, M., Chakrabarti, S.: Collection and recycle bin location-allocation problem in solid waste management: A review. Pollution 1(2), 175–191 (2015)
21. Rossit, D.G., Toutouh, J., Nesmachnow, S.: Exact and heuristic approaches for multi-objective garbage accumulation points location in real scenarios. Waste Management 105, 467–481 (2020)
22. Saif, Y., Rizwan, M., Almansoori, A., Elkamel, A.: Municipality solid waste supply chain optimization to power production under uncertainty. Computers & Chemical Engineering 121, 338–353 (2019)
23. Toth, P., Vigo, D.: The vehicle routing problem. SIAM (2002)
24. Vázquez Brust, A.: Ruteo de alta perfomance con OSRM. Rpubs by RStudio (may 2018), `https://rpubs.com/HAVB/osrm`